

Exercise 55

- (a) Find an equation of the tangent line to the curve $y = 2/(1 + e^{-x})$ at the point $(0, 1)$.
- (b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

Solution

A point on the tangent line is known, so all that's needed is its slope. Take a derivative of the given function

$$\begin{aligned} y' &= \frac{d}{dx} \left(\frac{2}{1 + e^{-x}} \right) = \frac{\left[\frac{d}{dx}(2) \right] (1 + e^{-x}) - \left[\frac{d}{dx}(1 + e^{-x}) \right] (2)}{(1 + e^{-x})^2} \\ &= \frac{(0)(1 + e^{-x}) - [(e^{-x}) \cdot \frac{d}{dx}(-x)] (2)}{(1 + e^{-x})^2} \\ &= \frac{-[(e^{-x}) \cdot (-1)] (2)}{(1 + e^{-x})^2} \\ &= \frac{2e^{-x}}{(1 + e^{-x})^2} \end{aligned}$$

and evaluate it at $x = 0$.

$$y'(0) = \frac{1}{2}$$

Therefore, the equation of the tangent line to $y = 2/(1 + e^{-x})$ at $(0, 1)$ is

$$y - 1 = \frac{1}{2}(x - 0).$$

Below is a graph showing the function and the tangent line.

